

Chapter 3 Study Materials

Math Notes

- Lesson 3.1.2 – Laws of Exponents
- Lesson 3.2.1 – Using Algebra Tiles to Solve Equations
- Lesson 3.2.2 – Multiplying Algebraic Expressions with Tiles
- Lesson 3.2.3 – Vocabulary for Expressions
- Lesson 3.2.4 – Properties of Real Numbers
- Lesson 3.3.1 – The Distributive Property
- Lesson 3.3.2 – Linear Equations From Slope and/or Points
- Lesson 3.3.3 – Using Generic Rectangles to Multiply

2. MAKING CONNECTIONS

Below is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

area	terms	Distributive Property
polynomial	expression	legal moves
generic rectangles	integers	dimensions
equation	product	algebra tiles
closed sets	sum	solution
solve	standard form	evaluate
binomial	length · width	base
exponent		



METHODS AND MEANINGS

MATH NOTES

Laws of Exponents

In the expression x^3 , x is the **base** and 3 is the **exponent**.

$$x^3 = x \cdot x \cdot x$$

The patterns that you have been using during this section of the book are called the **laws of exponents**. Here are the basic rules with examples:

Law	Examples
$x^m x^n = x^{m+n}$ for all x	$x^3 x^4 = x^{3+4} = x^7$ $2^5 \cdot 2^{-1} = 2^4$
$\frac{x^m}{x^n} = x^{m-n}$ for $x \neq 0$	$x^{10} \div x^4 = x^{10-4} = x^6$ $\frac{5^4}{5^7} = 5^{-3}$
$(x^m)^n = x^{mn}$ for all x	$(x^4)^3 = x^{4 \cdot 3} = x^{12}$ $(10^5)^6 = 10^{30}$
$x^0 = 1$ for $x \neq 0$	$\frac{y^2}{y^2} = y^0 = 1$ $9^0 = 1$
$x^{-1} = \frac{1}{x}$ for $x \neq 0$	$\frac{1}{x^2} = (\frac{1}{x})^2 = (x^{-1})^2 = x^{-2}$ $3^{-1} = \frac{1}{3}$

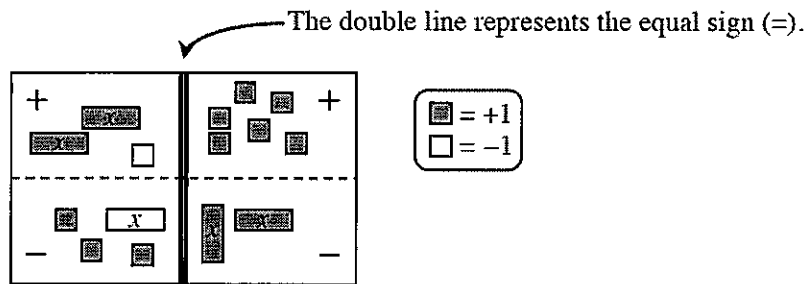


METHODS AND MEANINGS

MATH NOTES

Using Algebra Tiles to Solve Equations

Algebra tiles are a physical and visual representation of an equation. For example, the equation $2x + (-1) - (-x) - 3 = 6 - 2x$ can be represented by the Equation Mat below.



For each side of the equation, there is an addition and a subtraction region.

An Equation Mat can be used to represent the process of solving an equation. The “legal” moves on an Equation Mat correspond with the mathematical properties used to algebraically solve an equation.

“Legal” Tile Move

- Group tiles that are alike together.
- Flip all tiles from subtraction region to addition region
- Flip everything on both sides
- Remove zero pairs (pairs of tiles that are opposites) within a region of the mat
- Place or remove the same tiles on or from both sides
- Arrange tiles into equal-sized groups

Corresponding Algebra

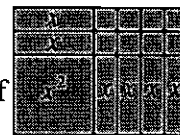
- Combine like terms.
- Change subtraction to “adding the opposite”
- Multiply (or divide) both sides by -1
- A number plus its opposite equals zero
- Add or subtract the same value from both sides.
- Divide both sides by the same value



METHODS AND MEANINGS

MATH NOTES

Multiplying Algebraic Expressions with Tiles



The area of a rectangle can be written two different ways. It can be written as a *product* of its width and length or as a *sum* of its parts. For example, the area of the shaded rectangle at right can be written two ways:

$$\underbrace{(x+4)}_{\text{length}} \underbrace{(x+2)}_{\text{width}} = \underbrace{x^2 + 6x + 8}_{\text{area}}$$

area as a product = area as a sum



METHODS AND MEANINGS

MATH NOTES

Vocabulary for Expressions

A mathematical **expression** is a combination of numbers, variables, and operation symbols. Addition and subtraction separate expressions into parts called **terms**. For example, $4x^2 - 3x + 6$ is an expression. It has three terms: $4x^2$, $3x$, and 6 . The **coefficients** are 4 and 3 . 6 is called a **constant term**.

A one-variable **polynomial** is an expression which only has terms of the form:

$$(\text{any real number}) x^{(\text{whole number})}$$

For example, $4x^2 - 3x^1 + 6x^0$ is a polynomial, so the simplified form, $4x^2 - 3x + 6$ is a polynomial.

The function $f(x) = 7x^5 + 2.5x^3 - \frac{1}{2}x + 7$ is a polynomial function.

The following are not polynomials: $2^x - 3$, $\frac{1}{x^2 - 2}$ and $\sqrt{x - 2}$

A **binomial** is a polynomial with only two terms, for example, $x^3 - 0.5x$ and $2x + 5$.



METHODS AND MEANINGS

MATH NOTES

Properties of Real Numbers

The legal tiles moves have formal mathematical names, called the **properties of real numbers**.

The **Commutative Property** states that when *adding* or *multiplying* two or more numbers or terms, order is not important. That is:

$$\begin{aligned} a + b &= b + a && \text{For example, } 2 + 7 = 7 + 2 \\ a \cdot b &= b \cdot a && \text{For example, } 3 \cdot 5 = 5 \cdot 3 \end{aligned}$$

However, *subtraction* and *division* are not commutative, as shown below.

$$\begin{aligned} 7 - 2 &\neq 2 - 7 && \text{since } 5 \neq -5 \\ 50 \div 10 &\neq 10 \div 50 && \text{since } 5 \neq 0.2 \end{aligned}$$

The **Associative Property** states that when *adding* or *multiplying* three or more numbers or terms together, grouping is not important. That is:

$$(a + b) + c = a + (b + c) \quad \text{For example, } (5 + 2) + 6 = 5 + (2 + 6)$$

$$(a \cdot b) \cdot c = a \cdot (b \cdot c) \quad \text{For example, } (5 \cdot 2) \cdot 6 = 5 \cdot (2 \cdot 6)$$

However, *subtraction* and *division* are not associative, as shown below.

$$(5 - 2) - 3 \neq 5 - (2 - 3) \quad \text{since } 0 \neq 6 \quad (20 \div 4) \div 2 \neq 20 \div (4 \div 2) \quad \text{since } 2.5 \neq 10$$

The **Identity Property of Addition** states that adding zero to any expression gives the same expression. That is:

$$a + 0 = a \quad \text{For example, } 6 + 0 = 6$$

The **Identity Property of Multiplication** states that multiplying any expression by one gives the same expression. That is:

$$1 \cdot a = a \quad \text{For example, } 1 \cdot 6 = 6$$

The **Additive Inverse Property** states that for every number a there is a number $-a$ such that $a + (-a) = 0$. A common name used for the additive inverse is the **opposite**. That is, $-a$ is the opposite of a . For example, $3 + (-3) = 0$ and $-5 + 5 = 0$

The **Multiplicative Inverse Property** states that for every nonzero number a there is a number $\frac{1}{a}$ such that $a \cdot \frac{1}{a} = 1$. A common name used for the multiplicative inverse is the **reciprocal**. That is, $\frac{1}{a}$ is the

reciprocal of a . For example, $6 \cdot \frac{1}{6} = 1$.



METHODS AND MEANINGS

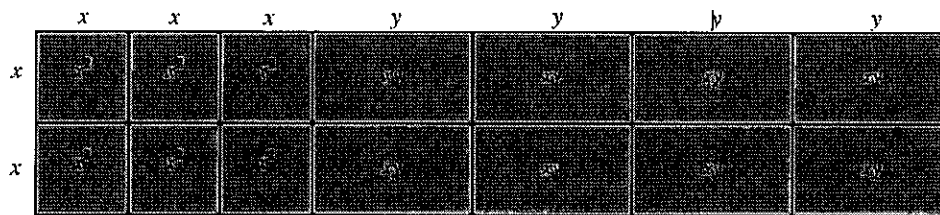
MATH NOTES

The Distributive Property

The **Distributive Property** states that for any three terms a , b , and c :

$$a(b + c) = ab + ac$$

That is, when a multiplies a group of terms, such as $(b + c)$, then it multiplies *each* term of the group. For example, when multiplying $2x(3x + 4y)$, the $2x$ multiplies both the $3x$ and the $4y$. This can be shown with **algebra tiles** or in a **generic rectangle** (see below).



$$2x \begin{array}{|c|c|} \hline 2x \cdot 3x & 2x \cdot 4y \\ \hline 3x & 4y \\ \hline \end{array}$$

$$2x(3x + 4y) = 2x(3x) + 2x(4y), \text{ simplifying results in}$$



The $2x$ multiplies each term.



METHODS AND MEANINGS

MATH NOTES

Linear Equations from Slope and/or Points

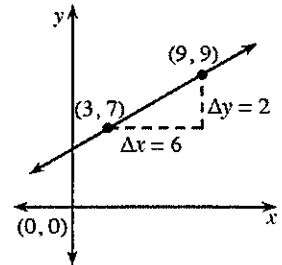
If you know the slope, m , and y -intercept, $(0, b)$, of a line, you can write the equation of the line as $y = mx + b$.

You can also find the equation of a line when you know the slope and one point on the line. To do so, rewrite $y = mx + b$ with the known slope and substitute the coordinates of the known point for x and y . Then solve for b and write the new equation.

For example, find the equation of the line with a slope of -4 that passes through the point $(5, 30)$. Rewrite $y = mx + b$ as $y = -4x + b$. Substituting $(5, 30)$ into the equation results in $30 = -4(5) + b$. Solve the equation to find $b = 50$. Since you now know the slope and y -intercept of the line, you can write the equation of the line as $y = -4x + 50$.

Similarly you can write the equation of the line when you know two points. First use the two points to find the slope. Then substitute the known slope and either of the known points into $y = mx + b$. Solve for b and write the new equation.

For example, find the equation of the line through $(3, 7)$ and $(9, 9)$. The slope is $\frac{\Delta y}{\Delta x} = \frac{2}{6} = \frac{1}{3}$. Substituting $m = \frac{1}{3}$ and $(x, y) = (3, 7)$ into $y = mx + b$ results in $7 = \frac{1}{3}(3) + b$. Then solve the equation to find $b = 6$. Since you now know the slope and y -intercept, you can write the equation of the line as $y = \frac{1}{3}x + 6$.



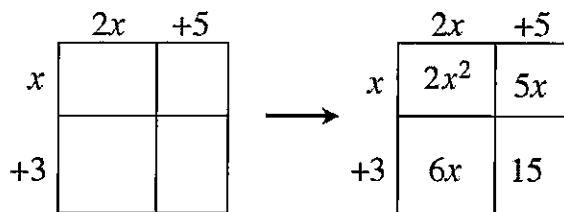


METHODS AND MEANINGS

MATH NOTES

Using Generic Rectangles to Multiply

A generic rectangle can be used to find products because it helps to organize the different areas that make up the total rectangle. For example, to multiply $(2x + 5)(x + 3)$, a generic rectangle can be set up and completed as shown below. Notice that each product in the generic rectangle represents the area of that part of the rectangle.



$$(2x + 5)(x + 3) = 2x^2 + 11x + 15$$

area as a product area as a sum

Note that while a generic rectangle helps organize the problem, its size and scale are not important. Some students find it helpful to write the dimensions on the rectangle twice, that is, on both pairs of opposite sides.

LAWS OF EXPONENTS**3.1.1 and 3.1.2**

In general, to simplify an expression that contains exponents means to eliminate parentheses and negative exponents if possible. The basic **laws of exponents** are listed here.

$$(1) \quad x^a \cdot x^b = x^{a+b}$$

$$\text{Examples: } x^3 \cdot x^4 = x^7; \quad 2^7 \cdot 2^4 = 2^{11}$$

$$(2) \quad \frac{x^a}{x^b} = x^{a-b}$$

$$\text{Examples: } \frac{x^{10}}{x^4} = x^6; \quad \frac{2^4}{2^7} = 2^{-3}$$

$$(3) \quad (x^a)^b = x^{ab}$$

$$\text{Examples: } (x^4)^3 = x^{12}; \quad (2x^3)^5 = 2^5 \cdot x^{15} = 32x^{15}$$

$$(4) \quad x^0 = 1$$

$$\text{Examples: } 2^0 = 1; \quad (-3)^0 = 1; \quad \left(\frac{1}{4}\right)^0 = 1$$

$$(5) \quad x^{-n} = \frac{1}{x^n}$$

$$\text{Examples: } x^{-3} = \frac{1}{x^3}; \quad y^{-4} = \frac{1}{y^4}; \quad 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

$$(6) \quad \frac{1}{x^{-n}} = x^n$$

$$\text{Examples: } \frac{1}{x^{-5}} = x^5; \quad \frac{1}{x^{-2}} = x^2; \quad \frac{1}{3^{-2}} = 3^2 = 9$$

$$(7) \quad x^{m/n} = \sqrt[n]{x^m}$$

$$\text{Examples: } x^{2/3} = \sqrt[3]{x^2}; \quad y^{1/2} = \sqrt{y}$$

In all expressions with fractions we assume the denominator does not equal zero.

For additional information, see the Math Notes box in Lesson 3.1.2. For additional examples and practice, see the Checkpoint 5A problems in the back of the textbook.

Example 1

$$\text{Simplify: } (2xy^3)(5x^2y^4)$$

$$\text{Reorder: } 2 \cdot 5 \cdot x \cdot x^2 \cdot y^3 \cdot y^4$$

$$\text{Using law (1): } 10x^3y^7$$

Example 2

$$\text{Simplify: } \frac{14x^2y^{12}}{7x^5y^7}$$

$$\text{Separate: } \left(\frac{14}{7}\right) \cdot \left(\frac{x^2}{x^5}\right) \cdot \left(\frac{y^{12}}{y^7}\right)$$

$$\text{Using laws (2) and (5): } 2x^{-3}y^5 = \frac{2y^5}{x^3}$$

Example 3

Simplify: $(3x^2y^4)^3$

Using law (3): $3^3 \cdot (x^2)^3 \cdot (y^4)^3$

Using law (3) again: $27x^6y^{12}$

Example 4

Simplify: $(2x^3)^{-2}$

Using law (5): $\frac{1}{(2x^3)^2}$

Using law (3): $\frac{1}{2^2 \cdot (x^3)^2}$

Using law (3) again: $\frac{1}{4x^6}$

Example 5

Simplify: $\frac{10x^7y^3}{15x^{-2}y^3}$

Separate: $\left(\frac{10}{15}\right) \cdot \left(\frac{x^7}{x^{-2}}\right) \cdot \left(\frac{y^3}{y^3}\right)$

Using law (2): $\frac{2}{3}x^9y^0$

Using law (4): $\frac{2}{3}x^9 \cdot 1 = \frac{2}{3}x^9 = \frac{2x^9}{3}$

Problems

Simplify each expression. Final answers should contain no parentheses or negative exponents.

1. $y^5 \cdot y^7$

2. $b^4 \cdot b^3 \cdot b^2$

3. $8^6 \cdot 8^{-2}$

4. $(y^5)^2$

5. $(3a)^4$

6. $\frac{m^8}{m^3}$

7. $\frac{12m^8}{6m^{-3}}$

8. $(x^3y^2)^3$

9. $\frac{(y^4)^2}{(y^3)^2}$

10. $\frac{15x^2y^5}{3x^4y^5}$

11. $(4c^4)(ac^3)(3a^5c)$

12. $(7x^3y^5)^2$

13. $(4xy^2)(2y)^3$

14. $\left(\frac{4}{x^2}\right)^3$

15. $\frac{(2a^7)(3a^2)}{6a^3}$

16. $\left(\frac{5m^3n}{m^5}\right)^3$

17. $(3a^2x^3)^2(2ax^4)^3$

18. $\left(\frac{x^3y}{y^4}\right)^4$

19. $\left(\frac{6x^8y^2}{12x^3y^7}\right)^2$

20. $\frac{(2x^5y^3)^3(4xy^4)^2}{8x^7y^{12}}$

21. x^{-3}

22. $2x^{-3}$

23. $(2x)^{-3}$

24. $(2x^3)^0$

25. $5^{1/2}$

26. $\left(\frac{2x}{3}\right)^{-2}$

Answers

- | | | |
|------------------------------|----------------------|-----------------------------|
| 1. y^{12} | 2. b^9 | 3. 8^4 |
| 4. y^{10} | 5. $81a^4$ | 6. m^5 |
| 7. $2m^{11}$ | 8. x^9y^6 | 9. y^2 |
| 10. $\frac{5}{x^2}$ | 11. $12a^6c^8$ | 12. $49x^6y^{10}$ |
| 13. $32xy^5$ | 14. $\frac{64}{x^6}$ | 15. a^6 |
| 16. $\frac{125n^3}{m^6}$ | 17. $72a^7x^{18}$ | 18. $\frac{x^{12}}{y^{12}}$ |
| 19. $\frac{x^{10}}{4y^{10}}$ | 20. $16x^{10}y^5$ | 21. $\frac{1}{x^3}$ |
| 22. $\frac{2}{x^3}$ | 23. $\frac{1}{8x^3}$ | 24. 1 |
| 25. $\sqrt{5}$ | 26. $\frac{9}{4x^2}$ | |

EQUATIONS ↔ ALGEBRA TILES**3.2.1**

An Equation Mat can be used together with algebra tiles to represent the process of solving an equation. For assistance with Lesson 3.2.1, see Lessons A.1.1 through A.1.9 in this *Parent Guide with Extra Practice*.

See the Math Notes box in Lesson A.1.8 (in the Appendix chapter of the textbook) and in Lesson 3.2.1 for a list of all the “legal” moves and their corresponding algebraic equivalents. Also see the Math Notes box in Lesson A.1.9 (in the Appendix chapter of the textbook) for checking a solution.

For additional examples and practice, see the Checkpoint 1 materials at the back of the textbook.

EQUATIONS ↔ ALGEBRA TILES**3.2.1**

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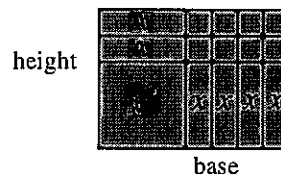
For additional examples and practice, see the Checkpoint 1 materials at the back of the textbook.

Two ways to find the area of a rectangle are: as a product of the (height)·(base) or as the sum of the areas of individual pieces of the rectangle. For a given rectangle these two areas must be the same, so **area as a product = area as a sum**. Algebra tiles, and later, generic rectangles, provide area models to help multiply expressions in a visual, concrete manner.

For additional information, see the Math Notes boxes in Lessons 3.2.2, 3.2.3, and 3.3.3. For additional examples and practice, see the Checkpoint 6B materials at the back of the textbook.

Example 1: Using Algebra Tiles

The algebra tile pieces $x^2 + 6x + 8$ are arranged into a rectangle as shown at right. The area of the rectangle can be written as the **product** of its base and height or as the **sum** of its parts.



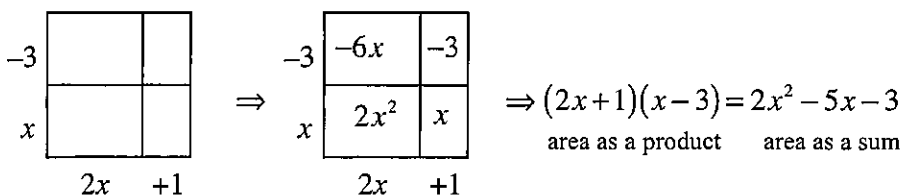
$$\underbrace{(x+4)}_{\text{base}} \underbrace{(x+2)}_{\text{height}} = \underbrace{x^2 + 6x + 8}_{\text{area}}$$

area as a product area as a sum

Example 2: Using Generic Rectangles

A generic rectangle allows us to organize the problem in the same way as the first example without needing to draw the individual tiles. It does not have to be drawn accurately or to scale.

Multiply $\underbrace{(2x+1)}_{\text{base}} \underbrace{(x-3)}_{\text{height}}$.



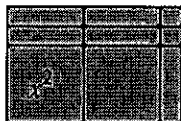
Problems

Write a statement showing area as a product equals area as a sum.

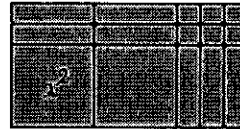
1.



2.



3.



4.

$$\begin{array}{|c|c|c|} \hline x & x^2 & 3x \\ \hline -5 & -5x & -15 \\ \hline \end{array}$$

$x \quad +3$

5.

$$6 \begin{array}{|c|c|} \hline 18y & -12x \\ \hline \end{array}$$

$3y \quad -2x$

6.

$$\begin{array}{|c|c|c|} \hline 3y & 3xy & 12y \\ \hline -2 & -2x & -8 \\ \hline \end{array}$$

$x \quad +4$

Multiply.

- | | | |
|--------------------|--------------------|--------------------|
| 7. $(3x+2)(2x+7)$ | 8. $(2x-1)(3x+1)$ | 9. $(2x)(x-1)$ |
| 10. $(2y-1)(4y+7)$ | 11. $(y-4)(y+4)$ | 12. $(y)(x-1)$ |
| 13. $(3x-1)(x+2)$ | 14. $(2y-5)(y+4)$ | 15. $(3y)(x-y)$ |
| 16. $(3x-5)(3x+5)$ | 17. $(4x+1)^2$ | 18. $(x+y)(x+2)$ |
| 19. $(2y-3)^2$ | 20. $(x-1)(x+y+1)$ | 21. $(x+2)(x+y-2)$ |

Answers

- | | | |
|----------------------------------|---------------------------------------|--------------------------|
| 1. $(x+1)(x+3) = x^2 + 4x + 3$ | 2. $(x+2)(2x+1) = 2x^2 + 5x + 2$ | |
| 3. $(x+2)(2x+3) = 2x^2 + 7x + 6$ | 4. $(x-5)(x+3) = x^2 - 2x - 15$ | |
| 5. $6(3y-2x) = 18y - 12x$ | 6. $(x+4)(3y-2) = 3xy - 2x + 12y - 8$ | |
| 7. $6x^2 + 25x + 14$ | 8. $6x^2 - x - 1$ | 9. $2x^2 - 2x$ |
| 10. $8y^2 + 10y - 7$ | 11. $y^2 - 16$ | 12. $xy - y$ |
| 13. $3x^2 + 5x - 2$ | 14. $2y^2 + 3y - 20$ | 15. $3xy - 3y^2$ |
| 16. $9x^2 - 25$ | 17. $16x^2 + 8x + 1$ | 18. $x^2 + 2x + xy + 2y$ |
| 19. $4y^2 - 12y + 9$ | 20. $x^2 + xy - y - 1$ | 21. $x^2 + xy + 2y - 4$ |

SOLVING EQUATIONS WITH MULTIPLICATION OR ABSOLUTE VALUE

3.3.1

To solve an equation with multiplication, first use the Distributive Property or a generic rectangle to rewrite the equation without parentheses, then solve in the usual way. For additional information, see the Math Notes box in Lesson 3.3.1. For additional examples and practice, see the Checkpoint 6B materials at the back of the textbook.

To solve an equation with absolute value, first break the problem into two cases since the quantity inside the absolute value can be positive or negative. Then solve each part in the usual way.

Example 1

Solve $6(x+2) = 3(5x+1)$

Use the Distributive Property.

$$6x + 12 = 15x + 3$$

Subtract $6x$.

$$12 = 9x + 3$$

Subtract 3.

$$9 = 9x$$

Divide by 9.

$$1 = x$$

Example 2

Solve $x(2x-4) = (2x+1)(x+5)$

Rewrite the equation using the Distributive Property on the left side of the equal sign and a generic rectangle on the right side.

$$2x^2 - 4x = \begin{array}{c} +5 \\ \begin{array}{|c|c|} \hline 10x & 5 \\ \hline x & 2x^2 & x \\ \hline \end{array} \\ 2x \quad + 1 \end{array}$$

$$2x^2 - 4x = 2x^2 + 11x + 5$$

Subtract $2x^2$ from both sides.

$$-4x = 11x + 5$$

Subtract $11x$ from both sides.

$$-15x = 5$$

Divide by -15 .

$$x = \frac{5}{-15} = -\frac{1}{3}$$

Example 3

Solve $|2x - 3| = 7$

Separate into two cases.

$2x - 3 = 7$ or $2x - 3 = -7$

Add 3.

$2x = 10$ or $2x = -4$

Divide by 2.

$x = 5$ or $x = -2$

Problems

Solve each equation.

- | | |
|--|---------------------------------------|
| 1. $3(c + 4) = 5c + 14$ | 2. $x - 4 = 5(x + 2)$ |
| 3. $7(x + 7) = 49 - x$ | 4. $8(x - 2) = 2(2 - x)$ |
| 5. $5x - 4(x - 3) = 8$ | 6. $4y - 2(6 - y) = 6$ |
| 7. $2x + 2(2x - 4) = 244$ | 8. $x(2x - 4) = (2x + 1)(x - 2)$ |
| 9. $(x - 1)(x + 7) = (x + 1)(x - 3)$ | 10. $(x + 3)(x + 4) = (x + 1)(x + 2)$ |
| 11. $2x - 5(x + 4) = -2(x + 3)$ | 12. $(x + 2)(x + 3) = x^2 + 5x + 6$ |
| 13. $(x - 3)(x + 5) = x^2 - 7x - 15$ | 14. $(x + 2)(x - 2) = (x + 3)(x - 3)$ |
| 15. $\frac{1}{2}x(x + 2) = \left(\frac{1}{2}x + 2\right)(x - 3)$ | 16. $ 3x + 2 = 11$ |
| 17. $ 5 - x = 9$ | 18. $ 3 - 2x = 7$ |
| 19. $ 2x + 3 = -7$ | 20. $ 4x + 1 = 10$ |

Answers

- | | | |
|--------------------------------|--|---------------------|
| 1. $c = -1$ | 2. $x = -3.5$ | 3. $x = 0$ |
| 4. $x = 2$ | 5. $x = -4$ | 6. $y = 3$ |
| 7. $x = 42$ | 8. $x = 2$ | 9. $x = 0.5$ |
| 10. $x = -2.5$ | 11. $x = -14$ | 12. all numbers |
| 13. $x = 0$ | 14. no solution | 15. $x = -12$ |
| 16. $x = 3$ or $-\frac{13}{3}$ | 17. $x = -4$ or 14 | 18. $x = -2$ or 5 |
| 19. no solution | 20. $x = \frac{9}{4}$ or $-\frac{11}{4}$ | |

Rewriting equations with more than one variable uses the same “legal” moves process as solving an equation with one variable in Lessons 3.2.1, A.1.8, and A.1.9. The end result is often not a number, but rather an algebraic expression containing numbers and variables.

For “legal” moves, see the Math Notes box in Lesson 3.2.1. For additional examples and more practice, see the Checkpoint 6A materials at the back of the textbook.

Example 1

Solve for y	$3x - 2y = 6$
Subtract $3x$	$-2y = -3x + 6$
Divide by -2	$y = \frac{-3x+6}{-2}$
Simplify	$y = \frac{3}{2}x - 3$

Example 2

Solve for y	$7 + 2(x + y) = 11$
Subtract 7	$2(x + y) = 4$
Distribute the 2	$2x + 2y = 4$
Subtract $2x$	$2y = -2x + 4$
Divide by 2	$y = \frac{-2x+4}{2}$
Simplify	$y = -x + 2$

Example 3

Solve for x	$y = 3x - 4$
Add 4	$y + 4 = 3x$
Divide by 3	$\frac{y+4}{3} = x$

Example 4

Solve for t	$I = prt$
Divide by pr	$\frac{I}{pr} = t$

Problems

Solve each equation for the specified variable.

- | | | |
|--|--|--|
| 1. Solve for y :
$5x + 3y = 15$ | 2. Solve for x :
$5x + 3y = 15$ | 3. Solve for w :
$2l + 2w = P$ |
| 4. Solve for m :
$4n = 3m - 1$ | 5. Solve for a :
$2a + b = c$ | 6. Solve for a :
$b - 2a = c$ |
| 7. Solve for p :
$6 - 2(q - 3p) = 4p$ | 8. Solve for x :
$y = \frac{1}{4}x + 1$ | 9. Solve for r :
$4(r - 3s) = r - 5s$ |

Answers (Other equivalent forms are possible.)

1. $y = -\frac{5}{3}x + 5$

2. $x = -\frac{3}{5}y + 3$

3. $w = -l + \frac{P}{2}$

4. $m = \frac{4n+1}{3}$

5. $a = \frac{c-b}{2}$

6. $a = \frac{c-b}{-2}$ or $\frac{b-c}{2}$

7. $p = q - 3$

8. $x = 4y - 4$

9. $r = \frac{7s}{3}$